Timed Literals & Exogenous Events

- Useful to represent **predictable exogenous events** that happen at known times, and cannot be influenced by the planning agent.

  For instance (using PDDL notation):

  ```
  (at 8 (open-fuelstation city1))
  (at 12 (not (open-fuelstation city1)))
  (at 15 (open-fuelstation city1))
  (at 19 (not (open-fuelstation city1)))
  ```

- Timed literals in the preconditions of an action impose **scheduling constraints** to the action:

  If `(refuel car city1)` has `over all` condition `open-fuelstation`, *it must be executed during the time window* $[8, 12]$ *or* $[15, 19]$.

  (Similarly for other types of action conditions)

DTP Constraints for PDDL2.2 Domains

- **Action ordering constraints**

  E.g., a must end (a^+) before the start of b (b^-): $a^+ \prec b^-$
  $a^+ \prec b^- \equiv a^+ - b^- \leq 0$

- **Duration Constraints**

  E.g., $(a^+ - a^- \leq 10) \wedge (a^- - a^+ \leq -10))$

- **Scheduling constraints** (in *compact* DTP-form):

$$\bigvee_{w \in W(p)} \Big(\big(a_{start} - a^- \leq -w^- \big) \wedge \big(a^+ - a_{start} \leq w^+ \big) \Big).$$

  If p `over all` timed condition with windows $W(p) = \{w_1, \ldots, w_n\}$
  (a_{start} is a special instantaneous action preceding all others)

  Note: we can compile all timed conditions of an action into a single `over all` timed precondition (with more time windows)

Temporally Disjunctive LA-graph

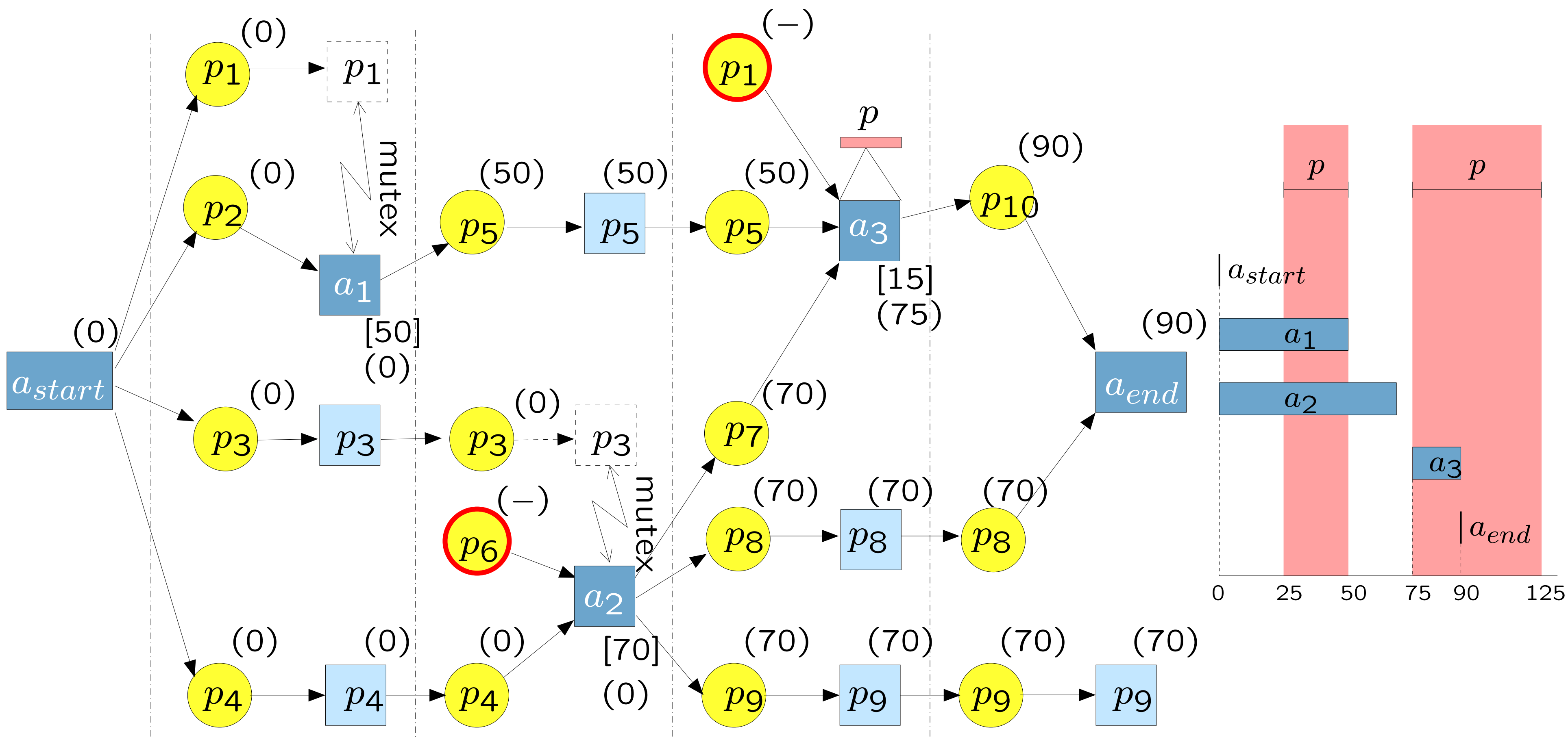
A **Temporally Disjunctive Action Graph (TDA-graph)** is a 4-tuple $\langle \mathcal{A}, \mathcal{T}, \mathcal{P}, \mathcal{C} \rangle$ where

- $\mathcal{A}$ is a linear action graph;
- $\mathcal{T}$ is an assignment of real values to the nodes of $\mathcal{A}$ (determined by solving the DTP $\langle \mathcal{P}, \mathcal{C} \rangle$)
- $\mathcal{P}$ is the set of time point variables representing the start/end times of the actions labeling the action nodes of $\mathcal{A}$;
- $\mathcal{C}$ is a set of ordering constraints, duration constraints and scheduling constraints involving variables in $\mathcal{P}$.

Propositional flaw: unsupported precondition node

Temporal flaw: action *un*scheduled by $\mathcal{T}$ ($\langle \mathcal{P}, \mathcal{C} \rangle$ is unsolvable)

Example of TDA-graph



$$
\mathcal{C} = \begin{cases} a_1^+ \prec a_3^-, \quad a_2^+ \prec a_3^-, \quad a_{start} \prec a_i^-, \quad a_i^+ \prec a_{end} \qquad (i = 1 \cdots 3) \\ a_1^+ - a_1^- = 50, \quad a_2^+ - a_2^- = 70, \quad a_3^+ - a_3^- = 15 \\ W_p = \{[25, 50), [75, 125)\} \Rightarrow a_3 \text{ during } [25, 50] \text{ or } [75, 125] \end{cases}
$$

Temporal values in a TDA-graph

- The DTP $\mathcal{D} = \langle \mathcal{P}, \mathcal{C} \rangle$ of a TDA-graph $\langle \mathcal{A}, \mathcal{T}, \mathcal{P}, \mathcal{C} \rangle$ represents a set Θ of STPs (unary constraints of $\mathcal{D}$ plus *at most one* disjunct for each disjunctive constraint)

- **Induced STP**: a satisfiable CSP in Θ

- **Complete induced STP**: an induced STP with exactly one disjunct (time window) for each disjunctive constraint

- **Optimal induced STP**: a complete induced STP with a solution assigning to a_{end} the minimum value over all solutions of every complete induced STP of $\mathcal{D}$

$\Rightarrow$ **Optimal schedule for $\mathcal{D}$ = $\mathcal{T}$-values**:
an optimal solution of an optimal induced STP of $\mathcal{D}$ for a_{end}.

Solving the DTP of a TDA-graph

Finding a solution for a DTP $\Longrightarrow$ solving a meta CSP:
[Stergiou & Koubarakis, Tsamardinos & Pollack, and others]

- *Meta variables*: constraints of the DTP
- *Meta variable values*: constraint disjuncts
- *Implicit meta constraint*: the values (constraint disjuncts) of the meta variables form a satisfiable STP

Solution of the meta CSP = complete induced STP of the DTP

In general NP-hard, but polynomial for the DTP of a TDA-graph:

Theorem: *Given the DTP $\mathcal{D}$ of a TDA-graph, deciding satisfiability of $\mathcal{D}$ and finding an optimal schedule for $\mathcal{D}$ (if one exists) can be accomplished in polynomial time.*