

Ragionamento Spaziale

Variabili: **regioni** dello spazio

Relazioni binarie per esprimere:

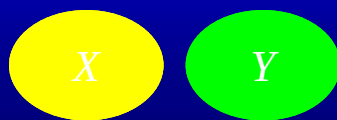
- La **posizione** relativa (destra, sinistra, nord, sud, ecc.)
- La **topologia** (disconnesse, attaccate, sovrapposte, ecc.)
- La **dimensione** relativa (qualitativa o quantitativa)
- La **distanza** relativa (qualitativa o quantitativa)
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Disgiunzioni di relazioni per esprimere conoscenza indefinita

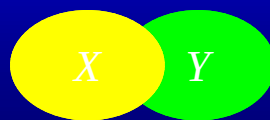
Ogni tipo di relazione ha associato un calcolo, che può essere combinato con altri.

1

RCC-5: 5 Basic Topological Relations Between Regions



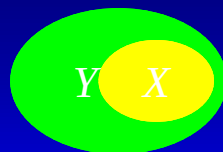
$X \text{ DC } Y$



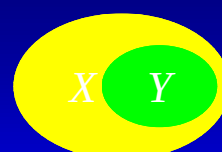
$X \text{ PO } Y$



$X \text{ EQ } Y$



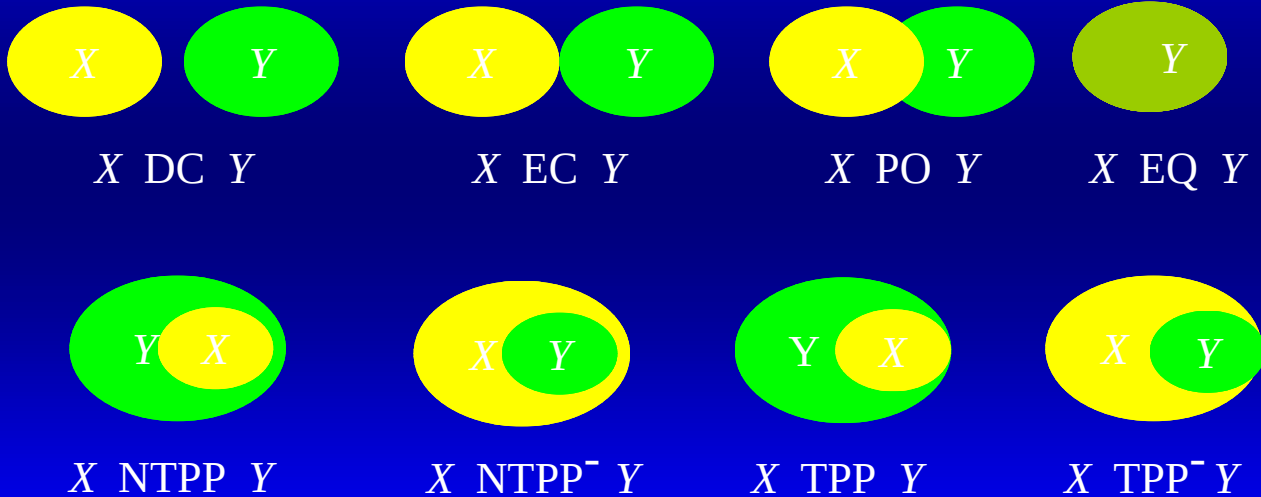
$X \text{ PP } Y$



$X \text{ PP}^- Y$

2

RCC-8: 8 Basic Topological Relations Between Regions



3

RCC-5: Tabella Composizione Relazioni di Base

	DC	PO	PP	PP-	EQ
DC	*	DC,PO,PP	DC,PO,PP	DC	DC
PO	DC,PO,PP	*	PP,PO	DC,PO,PP	PO
PP	DC	DC,PO,PP	PP	*	PP
PP-	DC,PO,PP-	PO,PP-	PO,PP,PP-	PP-	PP-
EQ	DC	PO	PP	PP-	EQ

4

RCC-5/8 Relations: Semantics, Syntax & Reasoning

- **Region variables** denote *regular, closed subsets* of a *topological space* (for instance R^2 or R^3)
- **Indefinite knowledge** expressed by *union of relations*, written as sets of base relations, e.g.

$X \{PO, EC\} Y$

→ 2^8 different relations for RCC-8 and 2^5 for RCC-5

- **Main inference task:** Given a set of topological formulae (a spatial CSP), is this set *satisfiable*?

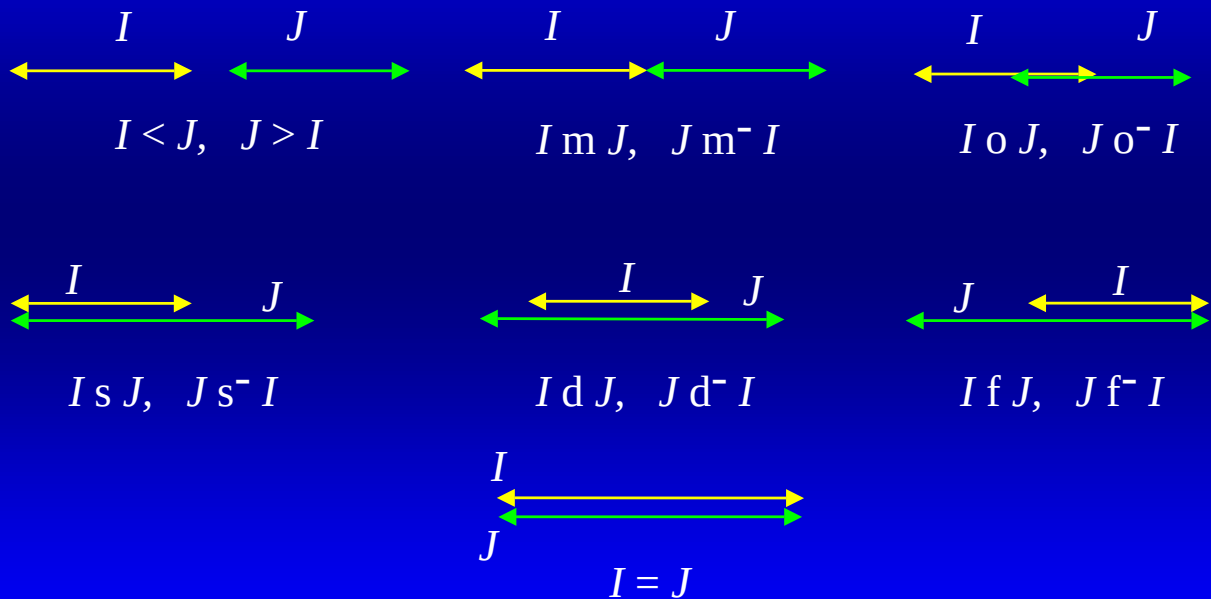
5

RCC-8: Computational Properties

- Satisfiability is **NP-complete (also for RCC-5!)**
- **3 maximal tractable fragments** containing the *universal* relation and all *base* relations: satisfiability is decidable by the *path-consistency algorithm* in $O(n^3)$ time [Renz & Nebel '99, Renz '99]
- These fragments can be extended with *qualitative relations* ($<, =, \leq, \neq$) *between the size of the regions*:
Satisfiability is **still decidable in $O(n^3)$ time** by the *Bipath-consistency algorithm* [Gerevini & Renz '98]

6

Allen's Basic Interval Relations



7

Interval Relations: Semantics, Syntax & Reasoning

- **Interval variables** denote pairs of rational numbers
- **Indefinite knowledge** is expressed by unions of relations (written as sets of base relations):

$I \{s,d,f\} J$

→ **8192** different relations

- Main **inference task**: Given a set of temporal formulae (a temporal CSP), is this set *satisfiable*?

8

Refinements & Scenarios For Qualitative Constraint Formulae

- Given a set of constraint formulae Θ (i.e., a CSP), we say Θ' is a **refinement** of Θ , if all formulae in Θ' are stronger (fewer base relations)

$$(I \{s,d,f\} J), (J \{<, >\} K) \rightarrow (I \{s\} J), (J \{>\} K)$$

$$(I \{PO,DC\} J), (J \{EC,PO\} K) \rightarrow (I \{PO\} J), (J \{PO\} K)$$

- A satisfiable CSP Θ is called **scenario** if there is a constraint with only *one base relation* between each pair of variables

$$(I \{s\} J), (J \{>\} K), (I \{>\} K)$$

$$(I \{PO\} J), (J \{PO\} K), (I \{DC\} K)$$

9

A Spatio-Temporal Constraint Calculus (Temporalizzazione di RCC-8 con IA)

- RCC-8 topological relationships between **regions** hold over a **temporal interval**
- Intervals are related using Allen's relations
- STCC CSP**: set of annotated RCC-8 constraint formulae, and Allen's constraints on their intervals

$$I: (X \{EC,DC\} Y), (Y \{TPP\} Z)$$

$$J: (X \{PO\} Y), (Y \{DC\} Z)$$

$$I \{<, m\} J$$

10

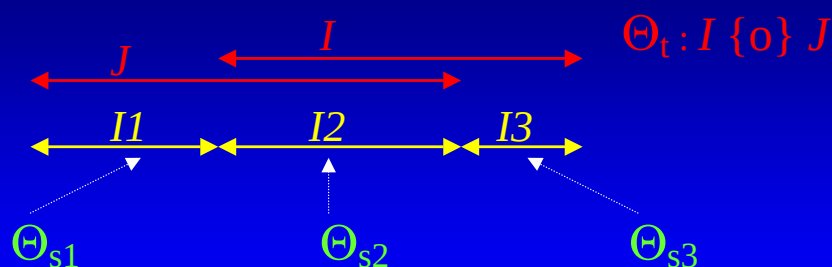
STCC CSPs: Semantics

- **Interpretation of a STCC CSP (Q, T, α) :**
 - Q : rational numbers
 - T : topological space
 - α : maps interval symbols to pairs of numbers from Q , and region symbols and rational numbers to non-empty, regular, closed subsets of T
- **Model (or Solution) of a STCC CSP:**
 Interpretation such that: all temporal constraints are satisfied, and each spatial formula annotated with I is satisfied at every point *between the endpoints of the interval* associated with I

11

Induced Scenario of a STCC CSP Θ

- Derive a temporal scenario Θ_t for the temporal CSP
- From Θ_t derive a total order of interval endpoints and the *induced* set of sub-intervals $I_1, I_2, \dots, I_k$
- Derive a spatial scenario Θ_{s_i} for each sub-interval I_i (Θ_{s_2} satisfies *both* the formulae annotated I and J)



12

Satisfiability of STCC CSPs: RISAT

RISAT is NP-complete (already noted by Bennett *et al.* '02)

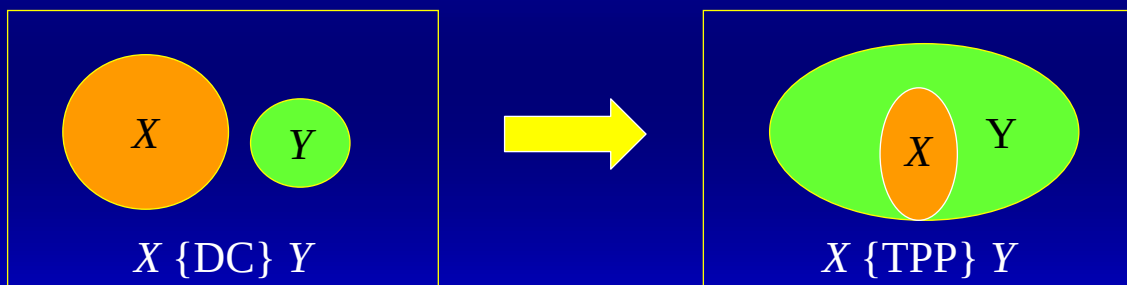
Theorem: RISAT is NP-hard even if the CSP contains only basic relations and the two universal relations.

Theorem: When the temporal relations form a scenario and the spatial relations are all elements of one tractable class, RISAT is polynomial.

13

Spatial Change

When spatial configurations change over time, are all changes meaningful?



- X and Y change their *shape*
- X and Y change their *size*
- The topological change is *not continuous*

14

The Size Persistence Constraint

The **size** of every region **persists** over time
(cannot be expressed in the STCC formalism)

I: $(X \{TPP\} Y)$ **J:** $(X \{EQ\} Y)$

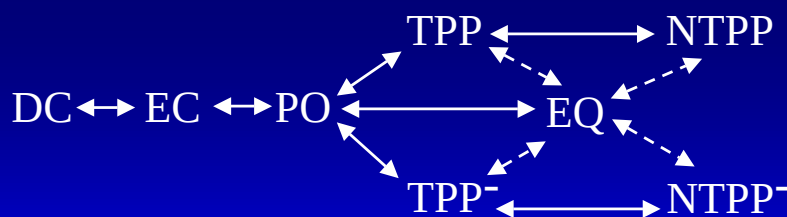
is not satisfiable regardless of the temporal relation between I and J

Theorem: ~~Is satisfiability still NP-complete?~~ $RISAT$ for a STCC CSP Θ with the size persistence constraint is NP-complete.

15

The Continuity Constraint

- Going from one Spatial Scenario to the next, the topological relations *must change continuously* (neighborhood structure [Freksa]):



- We may also want to preserve the size of all regions (no dashed lines)

16

STCC with the Continuity Constraint

- Problema computazionalmente ancora aperto!
Si *congettura* che decidere la soddisfacibilità sia un problema NP-completo, ma non e' ancora stato dimostrato completamente.

